Three Dimensional Geometry of the Heinike-Mikulicz Strictureplasty

By

Luka Pocivavsek M.D. PhD.¹², Efi Efrati PhD.²³, Ka Yee C. Lee PhD.²⁴, Roger D. Hurst M.D.¹

¹Department of Surgery, University of Chicago, Chicago, IL 60637 USA
²James Franck Institute, University of Chicago, Chicago, IL 60637 USA
³Department of Physics, University of Chicago, Chicago, IL 60637 USA
⁴Department of Chemistry, University of Chicago, Chicago, IL 60637 USA

Corresponding Author:
Roger D. Hurst
Associate Professor of Surgery
University of Chicago Pritzker School of Medicine
5841 S. Maryland Ave
MC-5093
Chicago, IL 60637
Telephone: (773) 702-6692
FAX: (773) 834-1995
Email: rhurst@surgery.bsd.uchicago.edu
Abstract:

Objective: The objective of this study is to assess the regional geometry and surface
topology (see comment in mail) with Heineke Mikulicz (HM) strictureplasty.

Background: The HM intestinal strictureplasty is commonly performed for the treatment of
stricturing Crohn's disease of the small intestine. This procedure shifts relatively normal
proximal and distal tissue to the point of narrowing and thus increases the luminal diameter. The
overall effect on the regional geometry of the HM strictureplasty, however, has not been
previously described in detail.

Methods: HM strictureplasties were created in latex tubing and cast with an epoxy resin.
The resultant casts of the lumens were then imaged using a computerized tomography. Utilizing
three-dimensional vascular reconstruction software, the cross-sectional areas were determined
and the surface geometry was examined.

Results: The HM strictureplasty, while increasing the lumen at the point of the stricture,
also results in a counterproductive luminal narrowing proximal and distal to the strictureplasty.
Within the model used, cross-sectional area was diminished 25 to 50% below baseline. This
effect is enhanced when two strictureplasties are placed in close proximity to each other.

Conclusions: The HM strictureplasty results in alterations in the regional geometry that
may result in a compromise of the lumen of the bowel proximal and distal to the location of the
strictureplasty. When two HM strictureplasties are created in close proximity to each other, care
should be undertaken to assure that the lumen of the intervening segment is adequate.
Introduction:

Crohn's disease is a chronic intestinal inflammatory condition of unknown etiology that typically affects the small intestine and colon. Inflammation from small bowel Crohn's disease is often multi-focal in nature and can give rise to multiple non-adjacent areas of localized stricturing. Patients with chronic obstructive symptoms from multifocal strictures are often treated with intestinal strictureplasty. The most commonly applied strictureplasty technique is the Heineke Mikulicz (HM) strictureplasty. {Yamamoto, 2007 #13} This procedure is named after the pyloroplasty technique from which it is derived. The procedure involves creating an anti--mesenteric incision along the long axis of the intestine. {Alexander-Williams, 1986 #94} This incision is centered over the focal area of narrowing and is extended into relatively normal tissue both proximal and distal to the stricture. This longitudinal incision is then closed in a transverse fashion. With this, the HM strictureplasty draws tissue from areas proximal and distal to the stricture to add to the circumference at the stricture site. Because tissues proximal and distal to the site are shifted, changes in the regional geometry away from the stricture itself are likely. A detailed analysis of the effects on the regional geometry of the HM strictureplasty has yet to be described. The following study was undertaken to provide an experimental model in which to study the geometry of intestinal strictureplasties and to elucidate the geometric character and regional effects of the HM strictureplasty.

Methods

To precisely and accurately study the geometry of a single HM strictureplasty as well as the interaction between pairs, we designed a robust elastic model. Latex tubing with 2 cm inner diameter and 0.15 cm wall thickness (McMaster-Carr, Elmhurst, IL) was used to model the baseline cylindrical intestinal geometry. The HM procedure was carried out on the tubing: linear
incisions were made along the long cylindrical axis, closed transversely using interrupted 4-0 Polysorb suture (Tyco, Norwalk, CT) taking care that the incision vertices were well approximated. The suture lines were made water tight by external application of silicone rubber 732 multi-purpose sealant (Dow Corning, Midland, MI). The post-procedure tubing was cast with EpoFix cold setting embedding epoxy resin (Electron Microscopy Sciences, Hartfield, PA). EpoFix requires no heating thus assuring the rubber mold would not deform during the casting process. A setting time of 36-48 hours was used at room temperature. To obtain geometric parameters, the epoxy casts were imaged with computerized tomography in a Phillips iCT256 scanner using 0.9 mm slice thickness, 0.045 mm slice increments, V=120 kV, I=37 mA, mAs=30 mAs. The digitalized imaging allowed for calculation and analysis of the cross-sectional area by commercial Philips 3D vascular re-construction software (see supplemental figure 1A). In short, a curved axis (dashed red line in supplemental figure 1A) was manually defined and cross-sectional slices perpendicular to this axis were re-constructed from the raw CT images. The cross-sectional areas were measured using the re-constructed perpendicular slices. For optimal re-construction, the curved axis was placed along the center of the un-deformed tube and made to follow a continuous arc from each end. An exception was made for models showing severe luminal narrowing (1cm and 2cm data in figure 5), where the curved axis was centered more toward the mesenteric surface where the inter-strictureplasty lumen was not completely compromised, again this was necessary to carryout an optimal reconstruction of the curved cast. Since the arc-length distance is shortest on the anti-mesenteric surface and longest along the mesenteric surface, there is an extensional factor of order one between the lengths. All of our linear length data (the x-axis in figures 2, 5, and 6) is presented un-scaled, as derived from the reconstructions. The epoxy casts were also imaged using a Nikon DX90 camera. The camera
was controlled externally by the Nikon Capture Control software. All data analyses were performed using Matlab (Mathworks, Natick, MA).

For the single strictureplasties, three enterotomy lengths were studied: 2, 3, and 4 cm. For the double strictureplasty models, the procedure was the same except that two linear enterotomies were initially created (2 and 3 cm enterotomies were studied). Care was taken so that the two incisions were along the same longitudinal axis. Furthermore, the separation between two strictureplasties was measured as the shortest linear distance between the incisions (see supplemental figure 1B). The separation distances studied ranged from 1 to 7 cm in 1 cm step sizes. To model the Michelassi strictureplasty, a 20 cm incision was made, the tube was then transected, beveled and sutured in the standard method.

Advanced mathematical analysis of the resultant geometry of the HM strictureplasty was undertaken by considering the outer most lumen layer as an idealized 2D surface and studying its intrinsic geometry. The intrinsic geometry of a surface consists of the collection of all distances between points as measured on the surface. [reference in comment 2] The connection between the intrinsic geometry of a surface and the configuration it assumes in space is not trivial and is provided by the Gaussian curvature; see supplemental material for further details.

In general, when an initially flat (or cylindrical) surface is cut and reconnected along straight lines the resulting intrinsic geometry remains flat almost everywhere and contains only conical defects in which Gaussian curvature is condensed in points. [same as above] The magnitude of a Gaussian curvature condensation can be identified with the opposite of the angle excess in the cone. A birthday-hat like structure, which is constructed by cutting out a wedge of head angle $\alpha$ from a disc and reconnecting the free edges of the disc, corresponds to a positive Gaussian curvature condensation of magnitude $\alpha$ at the vertex (see supplemental figure 2a).
Similarly if a flat disc is notched and an excess wedge of head angle $\alpha$ is connected to the cut sides then the resulting Gaussian curvature condensation is of magnitude $-\alpha$ (see supplemental figure 2b). Similarly if a flat disc is notched and a wedge of head angle $\beta$ is connected to the cut sides then the resulting Gaussian curvature condensation is of magnitude $-(\beta + \alpha)$ (see supplemental figure 2b). It is important to emphasize that the condensation of Gaussian curvature to a point does not mean that the geometry was changed only at a point; in the above examples a whole wedge of material was either introduced or eliminated in order to generate the desired Gaussian curvature condensation.

**Results:**

In this model, the HM strictureplasty, depending on the length of the enterotomy, increases the luminal cross-sectional area by 50 to 150% above the baseline lumen at the point of the strictureplasty (Fig 2). Regional distortions from the strictureplasty, however, generated a decrease in luminal cross-sectional area of 25 to 50% below baseline just proximal and distal to the strictureplasty site. This compromise of the residual lumen was dramatically increased when two stricturplasties were placed in close proximity of each other (Fig 5).

For a detailed analysis of the geometry of the HM strictureplasty, we began with the model of the single strictureplasty in isolation (figure 2A). We identify three locations of Gaussian curvature condensation. First, there is the small central region in which the enterotomy end-points ($a$ and $a'$ in figure 1) are sutured together. In our physical models, this corresponds to the midpoint along the suture line (black open circle in figure 2A bottom panel). Geometrically, the strictureplasty procedure in this point corresponds to taking two full circles and joining them along a radial cut as shown in figure 3A, where the centers of the circles map onto points $a$ and
a’, and the joined centers give rise to a surface geometry similar to that seen in figure 3B. This central point carries a negative Gaussian curvature condensation of $-2\pi$ (saddle-like structure). The ridges of the saddle are separated by four valleys (shaded in yellow in figure 2A) and also clearly seen in different projections of the mathematical models in figures 3D-F.

A second set of curvature condensation points flank the central $-2\pi$ region. These points arise from the transverse closure of the enterotomy at the ends of the suture line ($b$ and $b'$ in figure 1 and red open circles in figure 2A). Geometrically, the structure of each of these flanking corners corresponds to connecting the two radial lines in a semi-circle to generate a cone of Gaussian curvature condensation of $+\pi$. The fact that the Gaussian curvature condensations sum up to zero (two of $+\pi$ and one of $-2\pi$) is not coincidental and is associated with the fact that far from the strictureplasty-site the geometry remains unchanged. See supplementary material for further details. In summary, the geometry of the HM strictureplasty is set by the linear enterotomy and transverse closure that generates three points of curvature condensation: a central saddle like structure and two flanking cones.

The observed geometry of the single strictureplasty is strongly dominated by the $-2\pi$ Gaussian curvature condensation. We have tried cutting out the points of positive Gaussian curvature condensations, as well as introducing various cuts to the anti-mesenteric side, but these did not alter the geometry significantly. This conclusion is also supported by Figure 3 where an elastic model of the negative Gaussian curvature condensation alone (excluding both the cylindrical geometry and the positive Gaussian curvature condensation) Figure 3B, successfully reproduces the shape of the corresponding region in the HM strictureplasty as it appears in Figure 3C (also see supplemental movie 1).

We next investigate what impact this geometry has on luminal cross-sectional area of our
models. Figure 2B presents the CT derived cross-sectional areas $A$ as a function of curved distance along the post-strictureplasty model, i.e. arc length $\ell$. We center our data along the transverse suture line and define this plane as $\ell = 0$; furthermore, we use the un-deformed tube diameter $d$ as our internal length scale. In the case of all three enterotomy lengths (2, 3, and 4 cm), two regimes are immediately definable. Regime I, away from the enterotomy, and defined by $\ell \leq -d$ or $+d \leq \ell$, (outside the green box in figure 2B) where $A$ is equal to $A_0$ the area of the un-deformed cylinder, and the corresponding relative change in cross-sectional area $(A - A_0)/A_0$ is zero. Significant deformation fields induced by the strictureplasty geometry are confined to the vicinity of the enterotomy extending one tube diameter in each direction, $-d \leq \ell \leq +d$. We define this as regime II, (inside the green box in figure 2B). In this region both area dilation and area contracture are observed. Centrally located under the suture line is an area of strong dilation: $A(\ell = 0) = (1.5 - 3) \cdot A_0$. However this dilated region does not smoothly connect to the un-deformed cylinder of regime I, but rather is flanked by regions of area contracture both distally and proximally: $A(\ell = \pm d/2) = 1/2 \cdot A_0$. It is these flanking regions that make the re-connection to the un-deformed cylinder. The existence of strong dilation is not surprising as the procedure is successful in dilating strictured bowel. However, the strictureplasty literature seems to uniformly assume a smooth connection between dilated strictureplasty and un-deformed bowel. We leave the potential clinical implications of the more complex strictureplasty structure discovered by our experiments for the discussion section.

We note that the existence of both dilated and contracted regions within the strictureplasty is consistent with its saddle-like geometry discussed above. Figure 3 shows the geometric shapes generated by fusing two circles in different projections. The dilated region exists directly under the suture line and corresponds to the region around the horizontal ridge
where the circles were sutured together. The size of the area underneath the ridge clearly depends on its length, which is simply the length of the initial enterotomy in our models. Indeed, the data in figure 2B show that the dilation increases with enterotomy length. Moreover, figure 3E and 3F clearly show that just proximal and distal to the horizontal suture line, the sheet is pinched inwards. The four valleys radiating from the midpoint of the suture line (≈2π condensation point) drive this inward displacement. In fact, by comparing the cross-sectional images of our model strictureplasties in the contracture area (see figure 2B bottom-most images), it is easily appreciated that the structure is nearly triangular in agreement with the triangular opening seen in figure 3F. Geometrically, the degree of pinch-off is independent of circle radius or the length of the suture line, as long as there is sufficient length for the tube to close on the mesenteric side.

Again, our data are in agreement, showing that the degree of contracture is less sensitive to enterotomy size than the degree of dilation. In summary, we conclude that the dilation (50-150% increase in cross-sectional area) is simply driven by the transverse closure of the enterotomy; however, the strong condensation of negative Gaussian curvature that occurs during this closure induces regions of area contracture (~25-50% decrease relative to undeformed tubing).

The second part of our study focuses on how the global geometry and luminal area change as multiple strictureplasties are placed in one tube. Figure 4 shows images for a set of 2 cm and 3 cm enterotomies with enterotomy separation varying from 1 cm to 7 cm (at 1 cm intervals). Visually it is apparent that below a separation of 3 cm, a transition occurs. To more precisely characterize this transition, we follow the cross-sectional area of the different models using CT. The data in figure 5 can be divided into two regimes. A weak-interaction regime for \( \lambda > d \), where \( \lambda \) is the strictureplasty separation length (see supplemental figure 1B for further definition). In this regime, the two strictureplasties have the same geometry as in the single
enterotomy cases studied above: central area of dilation flanked by areas of contracture. Beyond each strictureplasty, the cross-sectional area returns to that of the un-deformed cylinder: \( A(\ell = 0) = A_0 \). The conclusion here is that if separated by at-least one tube diameter, the geometry of multiple strictureplasties is independent of one another. A rather dramatic transition occurs once the enterotomies are placed within one tube diameter: \( \lambda \leq d \). Within this strong-interaction regime, the two strictureplasties strongly interact causing a very severe collapse of the cross-sectional area between the two sites: \( A(\ell = 0) \approx 0 \). The nearly total collapse of the inter-strictureplasty area is far beyond the milder contracture encountered with single strictureplasties, where the decrease in area was on the order of 25-50% versus over 85% encountered here. We leave the discussion of clinical implications of such counterproductive luminal narrowing for below.

As detailed in the introduction, multiple strictures can surgically be treated with either several HM procedures or alternatively with the Michelassi iso-perastaltic strictureplasty. We studied one model of the Michelassi (see figure 6). Our data show that the procedure leads to a nearly four-fold increase in luminal area, consistent with the doubling of the luminal diameter. Furthermore, the beveling effect at the end-points releases some of the Gaussian curvature condensation. This likely plays a role in alleviating any proximal or distal contracture that would otherwise occur. Unlike the circle-to-circle geometry that is inherent in the Heineke-Mikulicz, the wedges presented in supplemental figure 2B and 2C capture the Michelassi end-point geometry more clearly.

Lastly, we carried out experiments on tubes of different thickness and diameters to better understand the above observed luminal collapse between two strictureplasties. A phase diagram of double enterotomy/strictureplasties as a function of tube thickness \( t \), tube diameter \( d \),
enterotomy length, and strictureplasty separation distance ($\lambda$) is given in supplemental figure 3. Briefly, three dimensionless parameters are defined: $\psi = \lambda/d$, $\alpha = \text{tube thickness}/\text{enterotomy length}$, and $\phi = \text{enterotomy length}/d$. And our data show that the criteria for placing two HM strictureplasties within the strong-interaction regime are $\psi < 1$, $\phi \geq 1$, and $\alpha \leq 0.1$.

**Discussion:**

With our model for intestinal strictureplasty, we found a 25-50% decrease in the normal residual cross-sectional area just proximal and distal to an isolated HM strictureplasty. When two strictureplasties are created in series and in close proximity to each other, the compromising affect on the lumen is dramatically increased. This additive effect becomes prominent when the strictureplasties are positioned within a distance equal to or less than the diameter of the normal un-distorted lumen. Given that the HM strictureplasty is designed to increase luminal diameter at the point of stricturing by shifting tissues normally located above and below the stricture, some degree of narrowing of the lumen proximally and distally could have been anticipated. Yet, the degree to which this happens, at least with our model, was surprising. The effect seen when two strictureplasties are placed in close proximity is concerning.

It is important to note that the distance between strictureplasties is different from the distance between the strictures themselves. The distance between strictureplasties is a function of the distance between the strictures and the length of the enterotomy that is utilized to create the strictureplasty. For example, if two focal strictures located 7 cm apart in a segment of intestine with a baseline diameter of 3 cm are managed with HM strictureplasties each performed
with a 4 cm enterotomy, the resultant strictureplasties would be 3 cm apart and thus within the range where significant luminal compromise of the segment between the strictureplasties may occur.

The safety and effectiveness of HM strictureplasties in the management of stricturing Crohn's disease of the small intestine has been well-established. {Dietz, 2002 #15; Dietz, 2001 #17; Fazio, 1993 #2; Hurst, 1998 #42; Yamamoto, 2007 #13} The technique is an effective means of alleviating the symptoms of chronic partial obstruction while at the same time preserving intestinal length and functional absorptive surface area. From the excellent short term results reported in multiple case series, it is reasonable to conclude that any luminal compromising that may occur in proximity to the HM strictureplasty is not likely to result in early postoperative obstructive symptoms. The long-term consequences on luminal narrowing, however, may pose an interesting issue. Much attention has been paid to the consequences that the post surgical residual intestinal lumen may have on the recurrence rates for Crohn's disease. Considerable literature has been devoted to how varying anastomotic techniques would affect the likelihood of recurrence. {Hashemi, 1998 #110; Ikeuchi, 2000 #112; Munoz-Juarez, 2001 #95; Tersigni, 2003 #109; Yamamoto, 1999 #107} So far no such consideration has been applied to strictureplasty techniques. It has been suggested that surgical techniques that result in diminished or compromised luminal cross-sectional areas may result in earlier recurrences of inflammation and/or symptoms from recurrent Crohn's disease. {Kono, 2011 #75; Munoz-Juarez, 2001 #95; Yamamoto, 1999 #107} Some have contended that stasis of luminal contents may generate or aggravate the inflammatory response. {Munoz-Juarez, 2001 #95; Poggioli, 1997 #90; Yamamoto, 1999 #107} Clinical observations have also suggested that alleviation of stasis may result in improvement in disease activity. {Poggioli, 1997 #90; Stebbing, 1995 #93} Even if
residual lumen size were to have no effect on the activity of inflammation it would seem likely that an already compromised lumen would more readily constrict to a critical diameter that leads to earlier development of obstructive symptoms.

It is interesting to note that some authors have noted that upon re-exploration for recurrent Crohn's disease in those patients who had undergone previous intestinal strictureplasty, the strictureplasties themselves are often free of recurrence. {Stebbing, 1995 #93; Milsom, 1999 #95} Recurrences, however, are commonly noted to be in the same general region of the initially treated disease. In other words, recurrences may not typically occur at the stricturplasty site, but rather in the regions proximal or distal to the previous site. It has also been reported that recurrences are higher when multiple strictureplasies are employed when compared to strictureplasties performed in isolation. {Greenstein, 2009 #79}

Surgeons with experience in treating Crohn's disease have advised against placing HM strictureplasties in close proximity. These recommendations are based upon concerns regarding tension on the suture lines and possible compromise of the blood flow to the tissues in between suture lines that are placed in close proximity. We believe this study provides additional reasons for concern when performing HM strictureplasties that are separated from each other by relatively short distances. Under such circumstances it may be advisable to use alternative techniques such as resection, the Finney strictureplasty, or the Michelassi strictureplasty. The Michelassi strictureplasty appears to be well suited for managing multiple strictures located in close proximity. Our model demonstrated that this technique resulted in dramatic increase in the lumen throughout the length of the strictureplasty without any compromise of the natural lumen on either the proximal or distal ends.
While the full elastic problem determining the equilibrium shape of the HM strictureplasty is intractable theoretically, we have given evidence that the luminal cross-sectional variation is dominated by the surface geometry generated by the HM strictureplasty. This both renders our results independent of tissue properties and calls for further analysis of the geometry of the HM strictureplasty. Through such analysis and modeling it may be possible to propose modifications to the surgical technique that could ameliorate the effects described. Hence, the detailed mathematical analysis is included in this manuscript.

The main limitation of this study is that the model is created from inanimate materials and thus it cannot compensate or predict the variables that may occur in living tissue, such as motility and contractility effects, remodeling, and tissue growth compensation. The model, however, does provide for the consistency and reproducibility necessary for accurate measurement and analysis. Additionally all the experiments were performed in tubes without focal stricturing. This was done for the sake of consistency, but the absence of a focal stricture should not affect the key observations made. Because the surgical procedure extends both proximally and distally beyond the strictured area the global hyperbolic geometry caused by negative Gaussian curvature condensation at the central point will be dominated by tissue properties at the ends of the enterotomy. These ends exist in normal tissue. While the stricture tissue could potentially impact how the flanking conical structures develop, these areas are not relevant to the geometric issues that are the focal point of this study.

In summary, our model suggests that the HM strictureplasty results in compromise of the lumen proximal and distal to the strictureplasty site. This effect is greatly increased when two strictureplasties are placed in close proximity to each other. Care should be undertaken when performing multiple HM strictureplasties to assure that the intervening lumen is adequate.
Supplementary material – text:

The Gaussian curvature.

At every point in a smooth curved surface one can identify two principal directions of curvature in which the curvature normal to the surface assumes its extremal values, $\kappa_1$ and $\kappa_2$. Keeping the surface geometry unaltered (introducing no in plane strain) one may bend a surface, causing the principal directions and the principal curvatures $\kappa_1$ and $\kappa_2$ to change. However, Gauss’ celebrated theorem states that if no strain is introduced then the product of the two principal curvatures, $K = \kappa_1 \cdot \kappa_2$, also called the Gaussian curvature, must remain unchanged. The Gaussian curvature therefore serves as a good proxy for the intrinsic geometric structure of a surface, but is also related to the shape it assumes in space. For example in a cylinder of radius $R$ one of the principal curvatures reads $1/R$ while the other (oriented along the axis) reads zero. Therefore the Gaussian curvature vanishes identically implying that in every non-straining bending deformation of the cylinder one of the principal curvatures must vanish.

The Gauss Bonnet theorem

The total integrated Gaussian curvatures in a given patch of area, $\Omega$, is related to the turning of its boundary, $\partial \Omega$, through the famous Gauss-Bonnet theorem:

$$\int \int_{\Omega} K dA = 2\pi - \oint_{\partial \Omega} k_g \, dS.$$  

Note that the integrated Gaussian curvature is dimensionless, and can be considered as an angle. Utilizing this theorem one may conclude that if a given surface changes its geometry (without leaving any holes open) in a confined region $D$, then outside of $D$ all geometric quantities are unchanged, thus the right hand side of the Gauss Bonnet theorem remains unchanged. It
therefore follows that the total integrated Gaussian curvature must also remain unchanged. This also demonstrates the converse claim: a Gaussian curvature condensation that does not average out to zero (by nearby opposite sign curvature condensations), changes the intrinsic geometry away from the curvature condensation.

When a point or a line are associated with an infinite Gaussian curvature such that the total integrated Gaussian curvature is finite, we say that the Gaussian curvature condensates. The finite thickness of every physical sheet always prevents such infinities from occurring, and regularizes them to finite (but large) values confined to a small (but not vanishing) area. As long as we study the system at a scale that is much larger than this regularization scale, we may ignore this small scale structure and consider these as having zero area. In this work we make use of the Gauss-Bonnet theorem to infer the geometric properties of regions in the surface without the necessity of identifying the details of the local geometry.

**Conical defects in flat sheets**

A flat sheet with a point Gaussian curvature condensation is often referred to as a conical defect \cite{MBA08}. Three such conical defects are plotted in supplementary figure 2. The magnitude of the curvature condensation can be inferred from the Gauss-Bonnet theorem. The collection of points which are a distance R from the vertex of the conical defect generates a circular line with a uniform turning rate of \( k_g = 1/R \). If a wedge of head angle \( \alpha \) is removed from a disc and the disc closed to form a birthday hat, then the new perimeter of the disc reads \( S = (2\pi - \alpha)R \). When replaced to the Gauss Bonnet formula we obtain for the integrated Gaussian curvature
\[ \int K \, dA = \alpha \]. For the introduction of a wedge (angle excess), these formulas still hold, only this time \( \alpha \) is negative.
Figures:

Figure 1: Heineke-Mikulicz strictureplasty procedure performed in a patient with focal stricturing. (A.) Linear incision is made along the anti-mesenteric boarder, extending proximally (a) and distally (a’) across the stricture into healthy bowel. (B. and C.) The incision is closed transversely with the approximation of vertex points a and a’, which initially were separated by the length of the incision. (D.) Completed Heineke-Mikulicz strictureplasty.
Figure 2: Models of single enterotomy strictureplasties of varying length. (A.) CT derived three-dimensional reconstructions of final Heineke-Mikulicz geometries generated from 2, 3, and 4cm linear enterotomies. The shading in the last set of images highlights the different geometric structures discussed in the main text. (B.) shows the relative cross-sectional areas \( \left( \frac{A - A_0}{A_0} \right) \) of the three models from distal to proximal ends and across the strictureplasty sites as a function of arc length \( \ell \). Two regimes are identified: I. \( +d \leq \ell \leq +d \) (outside the green box) where \( A \) is equal to that of the un-deformed cylinder and II. \( -d \leq \ell \leq +d \) (inside the green box) where \( A \) deviates strongly and represents the region most strongly affected by the HM procedure, in both cases \( d = 20 \text{mm} \) and is the diameter of the un-deformed tube. Regime II. can further be subdivided into a central area of strong dilation, where \( A(\ell = 0) \approx (1.5 - 3) \cdot A_0 \) and the degree of dilation increases proportionally with increasing enterotomy length, and flanking areas of contraction just proximal and distal to the point of dilation, where \( A(\ell = \pm d/2) \approx 1/2 \cdot A_0 \) and the degree of narrowing depends less strongly on enterotomy length.
Figure 3: The geometry of a single HM strictureplasty is dominated by the $-2\pi$ Gaussian curvature condensation at the center of the strictureplasty site. (A.) The enterotomy ends, marked $a$ and $a'$ in Fig. 1 can be considered as the centers of two identical circles whose radii are half the enterotomy length. Within this framework it is obvious how the suturing of the two circles one to the other generates a $2\pi$ angle excess which corresponds to a $-2\pi$ Gaussian curvature condensation. (B.) The configuration obtained by minimizing the elastic bending energy of the two connected discs forming a $-2\pi$ Gaussian curvature condensation. The suture lines are assumed to have no bending rigidity, and the discs are not allowed to self-intersect. (C.) The scanned 3 cm enterotomy length model (shown in Fig. 2A) with a region of distance $< 1.5$ cm around the central vertex marked in dark blue. See supplementary material for a 3D movie. (D.) Top, (E.) diagonal and (F.) side views of the elastic bending minimizing configuration. The triangular opening visible in (F.) accounts for the proximal and distal cross-section area decrease visible in Fig. 2B.
Figure 4: Cast models of two strictureplasties (2 cm and 3 cm enterotomies) separated by 2, 3, 4, 5, 6, or 7 cm. The inter-plasty separation is measured as the distance between the inner vertices of the two strictureplasties in the un-deformed tubes (see supplemental figure 1). The images clearly show that as two strictureplasties approach within one tube diameter (2cm), a strong change in global geometry occurs.
Figure 5: (A.) CT derived cross-sectional areas of model double strictureplasties as a function of enterotomy length, 2 cm (red data) and 3 cm (blue data), and strictureplasty separation (1 to 7 cm). The curves are offset for each separation distance and centered at the mid-point for clearer visualization. The baseline in each set is drawn in as the dashed gray horizontal line and corresponds to a cross-sectional area identical to the un-deformed tube $A_0$ (solid black bar corresponds to a 100% change in cross-sectional area). Two regimes are identified: a weak-interaction regime occurring for separation distances greater than one tube diameter (white background data in (A.) and representative set with CT cross-sectional images in (B.)) and a strong interaction regime when strictureplasties are placed within one tube diameter (yellow background data in (A.) and representative set with CT cross-sectional images in (C.)). Within the weak-interaction regime, the two strictureplasties sites have the same local structure as that of the single strictureplasties in figure 2: central dilation with flanking contractions; however, beyond each strictureplasty the cross-sectional area returns to baseline: $A(\ell = 0) \approx A_0$. Within the strong-interaction regime, the two strictureplasties strongly interact causing a very severe collapse of the cross-sectional area between the two sites: $A(\ell = 0) \approx 0$. 
Figure 6: Cross-sectional area data for a model Michelassi strictureplasty (A.) and the CT three-dimensional reconstruction of the model (B.). The data show central dilation by a factor of 4 over the un-deformed tube consistent with radius doubling in the reconstructed section. Importantly, the connection between the dilated region and the un-deformed (region indicated by black arrows) is smooth with no proximal or distal contracture as in the Heineke-Mikulicz.
Supplemental Figure 1: (A.) shows a representative three-dimensional reconstruction, the red dashed line is the manually defined central axis of the post-strictureplasty geometry. Mathematically, this manually defined axis is equivalent to the shortest central arc-length of the curved geometry that the tube takes on after strictureplasty. To obtain cross-sectional areas along the curved axis, the Phillips CT vascular reconstruction package is used (image on right in (A.) shows a representative 2D projection of the reconstruction). (B.) Schematic representation of two enterotomies (dashed red lines) and several ways of measuring separation distance: in our experiments, we defined the inter-strictureplasty separation as the distance between $a_1'$ and $a_2$ in the pre-strictureplasty state. This length is convenient because it is invariant under the strictureplasty deformation and independent of enterotomy length. We note that in the operating room this separation is often measured as the center-to-center distance between two strictures, which in our model corresponds to the center-to-center distance of the enterotomies. This length scale lacks invariance under deformation and is enterotomy length dependent thus making it less convenient for comparing sets of data like ours, however the two are easily relatable: $\lambda_{\text{models}} = \lambda_{\text{surgical}} - \text{enterotomy length}$. 
Supplemental Figure 2: Conical defects in thin flat sheets. Left column shows the underlying geometry presented by a flat disc in which a finite angle wedge is removed or introduced generating either a deficit or excess of angle at the vertex. Right column shows the four nodes elastic bending energy minimizer of the reconnected thin sheet as calculated in [reference]. Note that solutions with different number of nodes exist but are not consistent with the cylindrical geometry discussed here. (A.) A wedge of angle $\pi/4$ is removed, and the two free sides (bolded blue) are connected. The resulting shape is that of a cone, and is associated with Gaussian curvature condensation of $\pi/4$. (B.) An angle excess of $\pi/8$ is introduced by fitting a wedge of head angle $3\pi/8$ into a $\pi/4$ opening. The resulting shape is saddle-like and is associated with a negative Gaussian curvature condensation of $-\pi/8$. (C.) A $-\pi/2$ Gaussian curvature condensation. Note that the amplitude of the undulation is significantly larger. The corresponding triangular narrowing seen in Fig 3F. increases as the Gaussian curvature condensation becomes more negative.
Supplemental Figure 3: Phase diagram of double enterotomy/strictureplasties as a function of tube thickness ($t$), tube diameter ($d$), enterotomy length, and strictureplasty separation distance ($\lambda$). Three dimensionless order-parameters are defined: $\psi = \lambda/d$, $\alpha = \text{tube thickness/enterotomy length}$, and $\phi = \text{enterotomy length/d}$. The data points bounded by red rectangles correspond to models where the cross-sectional area between the two strictureplasties has collapsed as in figure 4C. These points are used to define a region of phase-space (shaded red rectangle in (A.), in the $\psi/\alpha$ projection plane (B.), and the $\psi/\phi$ projection plane (C.)) in which inter-strictureplasty luminal collapse is expected to occur. The different data points are as follows: I. solid black circles: $d=3$ mm, $t=1.5$ mm, enterotomy=10 mm, $\lambda=3$-10 mm, II. solid red circles: $d=6$ mm, $t=1.5$ mm, enterotomy=15 mm, $\lambda=3$-20 mm, III. solid blue circles: $d=10$ mm, $t=1.5$ mm, enterotomy=20 mm, $\lambda=5$-40 mm, IV. open black circles: $d=12$ mm, $t=1.5$ mm, enterotomy=20 mm, $\lambda=5$-40 mm, V. open blue circles: $d=20$ mm, $t=1.5$ mm, enterotomyA=10 mm, enterotomyB=15 mm, enterotomyC=20 mm, enterotomyD=17 mm $\lambda=10$ mm, VI. open red circles: $d=15$ mm, $t=3$ mm, enterotomyA=15 mm, enterotomyB=20 mm, enterotomyC=25 mm, $\lambda=15$ mm, VII. solid blue triangles – 2 cm enterotomy data presented in figure 4A, VIII. open blue triangles – 3 cm enterotomy data presented in figure 4A.